STABILITY CRITERION FOR STEADY-STATE BURNING OF POWDERS

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ABSTRACT: A model of a powder with a variable surface temperature
is considered. A stability criterion for the steady-state burning of pow-
ders at constant pressure is found on the assumption that all the pro-
cesses in the gas phase and the reaction layer of the condensed phase
are inertialess. It is shown that the stability region is determined by
two parameters: $K = (T_1 - T_0)(\partial \ln m/\partial T_0)\hat{p}$ and $r = (\partial T_1/\partial T_0)\hat{p}$ (here
T_1 is the surface temperature, T_0 is the initial temperature of the pow-
der, m is the mass burning rate, p is pressure). Burning is always stable
if $k < 1$. If $k > 1$, the burning process is stable only when $r > (k - 1)^2/$
$/k + 1$.

In the theory of burning of powders developed by Ya. B. Zel'do-
vich [1] it is assumed that the temperature T_1 of the condensed phase
is constant with change of pressure or initial temperature T_0 of the
powder. In such a model the steady-state burning regime at constant
pressure is stable if the quantity

$$k = (T_1 - T_0)\left(\frac{\partial \ln m}{\partial T_0}\right)_p$$

is less than unity.

By experiment it was found that a) the stability criterion $k < 1$
is not always satisfied [2, 3], and b) due to the presence of a chemical
reaction in the surface layer of the condensed phase, the surface tem-
perature changes as a function of pressure and initial temperature. In
this connection it has been asserted [4] that the form of the stability
criterion can be modified to take into account the change of surface
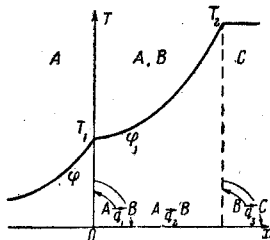temperature and the inertia of the reaction layer of the condensed
phase.

Fig. 1

Stability criteria have recently been derived [5, 6] on the as-
sumption that the chemical reaction zone in the condensed phase and
all the processes in the gas are inertialess. The authors of these studies
started out from a specific form of the kinetics of the chemical reac-
tions in the solid and gas phases.

In this paper, on the basis of a more general consideration of the
problem, it is shown that the region of stable burning (taking into ac-
count the inertia of the heated layer of the condensed phase only) is
determined by just two parameters—the above-mentioned Zel'dovich
parameter k and the derivative of surface temperature with respect to
initial temperature $r = \partial T_1/\partial T_0$ with $p = $ const. The values of k and r
can either be determined experimentally or found theoretically from
consideration of the specific form of the chemical reaction kinetics.

1. Two-stage model of powder burning. In the model considered
it is assumed that the conversion of the condensed fuel A into cumbus-
tion products C involves an intermediate gaseous state B, that is, A →
→ B → C. The first combustion stage comprises a decomposition pro-
cess A → B with release of heat $q_1 + q_2$, where q_1 is the heat release
in the condensed phase and q_2 is the heat released by the dispersed
particles of the condensed phase A upon decomposition in the gas

phase. Obviously, $q_1 + q_2$ is a constant, but q_1 and q_2 may vary in
dependence on the combustion conditions (pressure and initial tem-
perature of the powder). The intermediate gaseous products B, upon
being heated to combustion temperature T_2, react at this temperature
and yield combustion products C. We will assume that the activation
energy of the reaction B → C is quite large and that the chemical reac-
tion according to the gas combustion theory developed by Ya. B. Zel'-
dovich and D. A. Frank-Kamenetskii [7], proceeds virtually only at
the temperature T_2. In this case the mass burning rate depends only on
pressure and combustion temperature

$$m_2 = m_2(p, T_c). \tag{1.1}$$

The mass rate of the chemical reaction in the surface layer of
powder may depend only on the temperature distribution in the layer
and the heat release q_1. Assuming this layer to be inertialess, that is,
infinitely thin, we can write

$$m_1 = m_1(\varphi, \varphi_1, T_1, q_1) \qquad (\lambda\varphi - \lambda_1\varphi_1 = m_1 q_1). \tag{1.2}$$

Here φ and φ_1 are the internal and external temperature gra-
dients (the temperature distribution is shown in Fig. 1), λ and λ_1 are
the heat conductivities of the solid and gas phases, respectively.

At present, it is difficult to say what determines the heat re-
lease q_1 in the condensed phase; there is no theory that would give the
degree of dispersion. However, it can be said that the most general
dependence for q_1 has the form

$$q_1 = q_1(\varphi, \varphi_1, T_1, p). \tag{1.3}$$

To (1.1)–(1.3) we must add the condition that in the tempera-
ture region $T_1 < T < T_2$ all the processes are inertialess

$$m_1 = m_2 \tag{1.4}$$

and the obvious law of conservation of energy

$$mcT_1 + m(q_1 + q_2 + q_3) = \lambda\varphi + mc_1 T_2. \tag{1.5}$$

Here $q_3 = $ const is the thermal effect of the reaction B → C; here,
in accordance with (1.4), we have dropped the subscript on m, and c
and c_1 are the specific heats of powder and gas.

From the set of relationships (1.1)–(1.5) it is easy to establish
that the burning rate and surface temperature in the model are func-
tions of the gradient φ and pressure p only

$$m = m(\varphi, p), \qquad T_1 = T_1(\varphi, p). \tag{1.6}$$

The explicit form of these functions can be found theoretically
from a more specific consideration of the chemical reactions in the
condensed and gas phases and some particular assumption concerning
the dependence (1.3). The functional relations (1.6) can be deter-
mined experimentally by measuring the burning rate and surface tem-
perature at different pressures and initial temperatures of the powder
under stationary conditions. Then, using the stationary relation

$$\lambda\varphi = mc(T_1 - T_0) \tag{1.7}$$

we can transform the relations $m(T_0, p)$ and $T_1(T_0, p)$ to $m(\varphi, p)$ and
$T_1(\varphi, p)$.

2. Stability criterion for steady-state burning. We will investi-
gate the stability of steady-state burning for a model of a powder at
constant pressure. For stability it is necessary that small perturbations
of burning rate and temperature attenuate with time. On the other

hand, if the perturbations tend to increase, the retime is unstable. As usual, we superpose on the stationary solution small perturbations dependent on time as exp ωt and find equations for perturbations of burning rate, gradient φ, and surface temperature. Solving these equations, we determine the region of stability corresponding to the condition Re $\omega < 0$. Pressure is assumed to be constant and therefore all the derivatives are assumed to be taken with p = const.

From (1.6) we have two equations relating the perturbations of burning rate δm, gradient $\delta\varphi$, and surface temperature δT_1

$$\delta m = \frac{dm}{d\varphi}\,\delta\varphi, \qquad \delta T_1 = \frac{dT_1}{d\varphi}\,\delta\varphi. \qquad (2.1)$$

We obtain a third relation from the equation for heat conduction in the sold phase (the heated layer of powder is the only inertial region)

$$\rho c\,\frac{\partial T_\delta}{\partial t} = \lambda\,\frac{\partial^2 T_\delta}{\partial x^2} - m_\delta c\,\frac{\partial T_\delta}{\partial x}, \qquad (2.2)$$

where ρ is the powder density. We linearize the equation, substituting

$$T_\delta = T + \delta T e^{\omega t}, \qquad m_\delta = m + \delta m e^{\omega t},$$

where T and m are the steady-state temperature distribution and burning rate satisfying the equation

$$\lambda\,\frac{d^2 T}{dx^2} - mc\,\frac{dT}{dx} = 0, \qquad T\,|_{x=-\infty} = T_0, \qquad T\,|_{x=0} = T_1.$$

This equation, with the indicated boundary conditions taken into account, has the solution

$$T = T_0 + (T_1 - T_0)\exp\frac{mcx}{\lambda}.$$

In a linear approximation the equation for δT has the form

$$\rho c\omega\delta T = \lambda\,\frac{d^2\delta T}{dx^2} - mc\,\frac{d\delta T}{dx} - c^2(T_1 - T_0)\frac{m}{\lambda}\,\delta m\exp\frac{mcx}{\lambda}$$

with the boundary conditions

$$\delta T\,|_{x=-\infty} = 0, \qquad \frac{d\delta T}{dx}\,\Big|_{x=0} = \delta\varphi.$$

Hence we obtain the following relation between δm, δT_1, and $\delta\varphi$:

$$\frac{1+z}{2}\,\frac{\delta T_1}{T_1 - T_0} - \frac{\delta\varphi}{\varphi} + \frac{2}{1+z}\,\frac{\delta m}{m} = 0 \qquad (2.3)$$

$$\left(z = \left(1 + 4\,\frac{\lambda\rho\omega}{m^2 c}\right)^{1/2}, \ \operatorname{Re} z > 0\right).$$

Before solving system (2.1)–(2.3), we express the derivatives in (2.1) in terms of the experimentally determined values

$$k = (T_1 - T_0)\frac{d\ln m}{dT_0}, \qquad r = \frac{dT_1}{dT_0}.$$

From (1.7) we have

$$\frac{d\ln\varphi}{dT_0} = \frac{d\ln m}{dT_0} + \frac{d\ln(T_1 - T_0)}{dT_0} \qquad \text{or} \qquad (T_1 - T_0)\frac{d\ln\varphi}{dT_0} = k + r - 1.$$

Therefore

$$\frac{d\ln m}{d\ln\varphi} = \frac{k}{k+r-1}, \qquad \frac{1}{T_1 - T_0}\frac{dT_1}{d\ln\varphi} = \frac{r}{k+r-1}. \qquad (2.4)$$

Equations (2.1) can be written in the form

$$\frac{\delta m}{m} - \frac{k}{k+r-1}\,\frac{\delta\varphi}{\varphi} = 0, \qquad (2.5)$$

$$\frac{\delta T_1}{T_1 - T_0} - \frac{r}{k+r-1}\,\frac{\delta\varphi}{\varphi} = 0.$$

The solvability condition for system (2.3), (2.5) gives

$$\begin{vmatrix} \dfrac{1+z}{2} & -1 & \dfrac{2}{1+z} \\[2mm] 0 & \dfrac{k}{k+r-1} & -1 \\[2mm] 1 & -\dfrac{r}{k+r-1} & 0 \end{vmatrix} = 0,$$

or $\quad z = \dfrac{k-1 \pm \sqrt{(k-1)^2 - (2k-r+2)\,r}}{r}.$

It can be seen that all real systems have k > 0 and r > 0; r cannot significantly exceed unity and therefore we will limit ourselves to an investigation of this region only (Fig. 2). The parabola $(k-1)^2 = (2k - r + 2)r$ divides the plane (kr) into regions of real (below curve i) and complex (above i) z. As pointed out above, the region of unstable combustion is determined by the conditions

$$\operatorname{Re} z > 0, \quad \operatorname{Re}\omega \sim \operatorname{Re}(z^2 - 1) > 0.$$

In the region of real z we have k < 1 and there are no solutions at all for z > 0. If z is complex, Re z > 0 and Re $(z^2 - 1) < 0$ only when $r < (k-1)^2/(k+1)$.
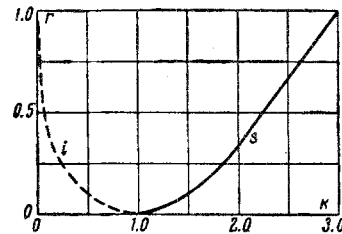

Fig. 2

Thus, burning is always stable if k < 1; when k > 1 the steady-state burning retime is stable only when

$$r > \frac{(k-1)^2}{k+1}.$$

In Fig. 2 the curve $r = (k-1)^2/(k+1)$ is denoted by the letter s. If r = 0, then as in the Zel'dovich theory the limit of stability is attained at k = 1.

In [5, 6] stability criteria were derived for some specific relationships between the reaction rates in the solid and gas phases. It is easy to confirm that the stability criteria for all five models considered in these studies follow from the result obtained above. In each specific case this confirmation requires computation of the values of k and r.

3. Conclusion. Two comments will be made. Since in the model considered the burning rate and surface temperature under nonstationary conditions are determined only by pressure and the temperature gradient at the surface of the solid phase, the theory of nonstationary phenomena during the burning of powders with change of surface temperature taken into account, can be developed in the same way as for the Zel'dovich model [1, 2, 8–10]. To be sure, as complication arises: in addition to knowing the functions $m(\varphi, p)$ it is necessary to know the dependence $T_1(\varphi, p)$.

The second comment applies to the stability region when k > 1. As pointed out in [5], in this region there may be oscillatory powder burning regimes (the frequency ω is complex). In particular, on the curve s the temperature and burning rate of a powder that has somehow departed from the steady-state regime oscillate without damping about their steady-state values. It is possible that in this region effects may be observed which are associated with the presence of a natural frequency in the system (for example, a resonance dependence of the burning rate on the frequency of the sonic wave incident on the surface of the powder).

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